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Plasma wave mediated attractive potentials: a prerequisite for electron compound formation

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Abstract. Coagulation of electrons to form macro-electrons or compounds in high temperature plasma is not generally expected to occur. Here we investigate, based on earlier work, the possibility for such electron compound formation (non-quantum "pairing") mediated in the presence of various kinds of plasma waves via the generation of attractive electrostatic potentials, the necessary condition for coagulation. We confirm the possibility of production of attractive potential forces in ion- and electron-acoustic waves, pointing out the importance of the former and expected consequences. While electron-acoustic waves presumably do not play any role, ion-acoustic waves may potentially contribute to formation of heavy electron compounds. Lower-hybrid waves also mediate compound formation but under different conditions. Buneman modes which evolve from strong currents may also potentially cause non-quantum "pairing" among cavity-/hole-trapped electrons constituting a heavy electron component that populates electron holes. The number densities are, however, expected to be very small and thus not viable for justification of macro-particles. All these processes are found to potentially generate cold compound populations. If such electron compounds are produced by the attractive forces, the forces provide a mechanism of cooling a small group of resonant electrons, loosely spoken, corresponding to classical condensation.

Keywords. Space plasma physics (magnetic reconnection; wave–particle interactions; waves and instabilities)

1 Introduction

Plasmas consist of equal numbers of electrons and ions forming quasi-neutral fluid-like matter at temperatures sufficiently high for maintaining ionisation and with very large numbers of electrons populating the Debye sphere. At such high temperatures electrons and ions are mutually well separated located at instantaneous distances $L_N \sim N^{-1/3}$, where $N \equiv N_e = N_i$ is the average plasma density in a singly charged plasma. The electrostatic Coulomb fields of the naked electric charges are confined to Debye spheres of radius λ_D by the collective effect of the many particles of opposite charge passing around at their average tangential speeds within radial distances $r \lesssim \lambda_D$. The geometric shape of the Debye spheres is very close to a sphere, deviating from it only in very strong magnetic fields and for very high plasma flow speeds. Outside the Debye sphere the residual particle field decays exponentially while contributing to a thermal fluctuation background field. From a particle point of view each of the plasma particles is a charged Fermion. In classical plasmas at the high plasma temperatures the spin has no importance, and the fermionic property of the particles plays no role. In quantum plasmas, which for obeying quantum properties must be dense, this property is rather important. For, when two electrons form pairs, the spins add up and the pair becomes a Boson of either zero or integer spin. Many pairs can occupy the same energy level and, altogether, tend to condensate in the lowest energy level permitted by sufficiently low temperatures (cf., however, the Appendix). This property is very well known from solid state physics (cf. e.g. Fetter and Walecka, 1971; Huang, 1987).

Pairing is not expected under any normal plasma conditions. However, when a plasma wave passes across the non-quantum plasma, the dielectric properties of the plasma change. Electrons assuming a relative velocity with respect to the phase velocity of the plasma wave finding themselves exposed to the dielectric polarisation which adds to the Debye screening that compensates for the naked particle charge. Such electrons may evolve an attractive electrostatic inter-particle force acting on its neighbour electrons, an effect different from classical wave trapping in the wave potential trough that causes wave saturation and other nonlinear effects like solitons and holes. The attractive forces are direct current (dc) forces. Experienced by two electrons of approximately same velocity, they bind these together to form binary compounds, that is, classical "pairs". Experienced by many electrons of same velocity, they may form large electron compounds the nature of which is that of massive macro-particles of same charge-to-mass ratio e/m being sufficiently strongly correlated to behave dynamically like one single particle. Below we demonstrate that it is not the electrons themselves which coagulate but their Debye clouds such that the repulsive forces between the coagulating electrons do grow only very weakly with the number n_{com} of coagulations.

Depending on the number n_{com} of particles in the newly formed conglomerate being odd or even, macro-electrons in principle behave like Fermions or Bosons. At very low temperatures Bosons are known to condensate to form a dense population. At the high plasma temperatures, trivially implying thermal wavelengths $\lambda_T = \sqrt{2\pi\hbar^2/m_{\text{e}}T} \approx 10^{-9}\sqrt{1\,\text{eV}/T}\,m$ of the order of atomic scales, such effects are inhibited (for another strong counter argument see the Appendix). Classical coagulation of particles caused by attractive inter-particle forces is, however, permitted.

In the following we show that, under certain conditions in high temperature plasmas, attractive potentials between neighbouring electrons are indeed produced as is schematically shown in Fig. 1. They are a necessary condition for subsequent coagulation of electrons. The intention is to find out whether it provides a natural mechanism justifying the still unavoidable assumption of macro-particles in numerical particle-in-cell simulations.

The finding is that, though electrons may possibly coagulate, their numbers will in all cases remain very small being just of the order of a fraction of the resonant particles which are a small fraction of the plasma population only, too small to be taken as justification of the assumption of macro-particles.

The conditions for generation of attractive electrostatic potentials in the presence of plasma waves are derived as a necessary condition for coagulation. The numbers of particles in each of the electron compounds, the compound densities and temperatures, require studying the sufficient conditions for coagulation for each kind of waves, which requires knowledge of the wave spectrum.[1]

2 Generation of wave-mediated attractive potentials

The method of calculating the potential around a test charge in plasma was explicated sixty years ago (Neufeld and Ritchie, 1955). Thirty years later it was revived (Nambu and Akama, 1985) to include the effect of plasma waves and was used in this form to suggest the coagulation of particles in the presence of dust in plasmas in order to explain the formation of dust structure, which for some while became an industry (cf. e.g. Shukla and Melandsø, 1997; Shukla et al., 2001; Nambu and Nitta, 2001, and references therein).

Following Neufeld and Ritchie (1955), the general expression for the electrostatic potential $\Phi(x,t)$ is obtained from Poisson's law $\nabla^2\epsilon(x,t)\Phi(x,t) = -qN(x,t)/4\pi\epsilon_0$. The density of a test particle traversing the plasma with velocity v and charge $q = q_t$ is $N_t = (2\pi)^3\delta(r)$. In Fourier space this yields for the potential (cf. Neufeld and Ritchie, 1955; Sitenko, 1967; Krall and Trivelpiece, 1973; the latter two for textbook presentations)

$$\Phi(x,t) = -\frac{q_t}{8\pi^2\epsilon_0}\int \mathrm{d}k\mathrm{d}\omega\,\frac{\delta(\omega - k\cdot v)}{k^2\epsilon(k,\omega)}\mathrm{e}^{ik\cdot r} \tag{1}$$

Here $r = x - vt$ is the distance between the location $x' = vt$ of the particle and the reference point of measurement of the potential disturbance. $\omega(k)$ is the frequency of a spectrum of plasma wave eigenmodes, presumably present in the plasma as background noise or wave excitations, with wave number k. The function $\epsilon(k,\omega)$ is the complete dielectric plasma response function[2] experienced by the particle including the disturbance caused by the moving charge of the test particle. (One may note that the δ function in the numerator can be used to replace the wave-number component parallel to the particle velocity in the Fourier exponential $\exp(ik\cdot r)$ reducing it to an ω integration.)

The problem as seen from the test charge is spherically symmetrical. Thus it makes sense to formulate it in spherical coordinates r, k, Ω_r, Ω_k both in wave number and real space, with conventional notation for the angular volume elements. Choosing an expansion into spherical harmonics (as done in

[1]We thank the referees for reminding us that generation of attractive potentials is merely the necessary condition for coagulation. Real coagulation will take place only if a number of sufficient conditions is also met, a point that will be further discussed in the Discussion and Conclusions section.

[2]It may be important to remark that in Poisson's law no assumption is made about the linearity or nonlinearity of $\epsilon(x,t)$ which, hence, in principle is the general response function including all electrostatic nonlinearities, which contribute to the dielectric properties of the plasma. Thus in full generality one should rather write $\epsilon[k,\omega,\Phi(k)]$. In discussing the Buneman mode later, we will in passing make use of this more general form.

Neufeld and Ritchie, 1955) one has for the exponential factor

$$e^{ik\cdot r} = 4\pi \sum_l \sum_m i^l \sqrt{\frac{\pi}{2kr}} J_{l+\frac{1}{2}}(kr) Y_l^{m*}(\Omega_k) Y_l^m(\Omega_r). \qquad (2)$$

The notation for the spherical harmonics $Y_l^m(\Omega_k), Y_l^m(\Omega_r)$ is again conventional in wave number and real space, and the $*$ indicates the conjugate complex version of the azimuthal exponentials $\exp(im\phi_k)$. The k, r dependence is taken care of by the half integer Bessel functions $J_{l+1/2}(kr)$.

The response function $\epsilon[k, \omega(k)]$ is a function of frequency and wave number and is taken in the electrostatic limit. In the above representation it is a scalar function. When electromagnetic contributions or an external magnetic field would have to be taken into account, it becomes a tensor. Only its longitudinal part $\epsilon_L = k \cdot \epsilon \cdot k/k^2$ enters the expression for the potential, however. In addition, one would have to consider a variation of the vector potential caused by the test particle, if transverse waves are included.

In the absence of the latter, it is well known that the potential of the test particle, in our case an electron of elementary charge $q = -e$, consists of its Coulomb potential $\Phi_C(r) = -q/4\pi\epsilon_0 r$, which will be Debye screened by the plasma particles becoming $\Phi_D(r) = \Phi_C(r)\exp(-r/\lambda_{\rm D})$, and a disturbance caused by the reaction of the plasma eigenmodes to the presence of the charge – the eigenmodes that are either present or are amplified by the moving test charge for which the plasma appears as a dielectric to whose normal modes the charge couples. These eigenmodes contribute via adding each of them to the vacuum dielectric constant its particular susceptibilities $\chi_{\rm s}(\omega, k)$, where s is the index identifying the particle species which responds to the eigenmodes. Hence, with $s = $ e, i for electrons and ions, respectively:

$$\epsilon(k, \omega) = 1 + (k\lambda_{\rm D})^{-2} + \chi_{\rm e}(k, \omega) + \chi_{\rm i}(k, \omega) \qquad (3)$$

Independent of the wave modes, the Debye term on the right is included here in order to account for the presence of the point like test charge. In a non-magnetised plasma the susceptibilities assume the form

$$\chi_{\rm s}(k, \omega) = (k\lambda_{\rm Ds})^{-2}[1 + \zeta_{\rm s} Z(\zeta_{\rm s})], \qquad (4)$$
$$\zeta_{\rm s} = (\omega - k \cdot u_{\rm s})/kv_{\rm s}.$$

$\lambda_{\rm Ds}$ is the Debye length of species s, $Z(\zeta_{\rm s})$ the plasma dispersion function, $v_{\rm s}$ the thermal speed of species s, and $u_{\rm s}$ is a possible bulk streaming velocity of species s which, for our application, will for simplicity be put to zero but has to be retained both for streaming and electric currents $\mathbf{J} = \sum_{\rm s} q_{\rm s} N_{\rm s} u_{\rm s}$.

One should note that Poisson's law is quite general holding for both linear and nonlinear interactions. Restriction to linear response functions only implies small disturbances caused. For large nonlinear disturbances the linear response function would have to be replaced by its nonlinear counterparts.

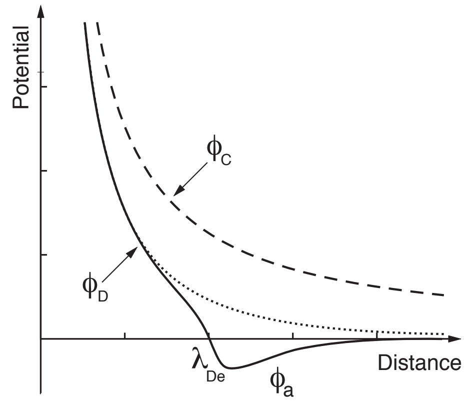

Figure 1. Schematic comparison between the dc Coulomb ($\Phi_{\rm C}$), Debye ($\Phi_{\rm D}$) and plasma wave modified ($\Phi_{\rm a}$) potentials for the case when the modified potential evolves an attractive domain. In all cases considered the potential is repulsive inside the Debye sphere at distances $\leq \lambda_{\rm De}$. In the presence of (electrostatic) plasma waves investigated in this paper the dc potential $\Phi_{\rm a}$ outside but close to a distance $\lambda_{\rm De}$ develops a region where it becomes negative. At large distances the effect of the plasma wave decreases rapidly.

These expressions have been partially analyzed in the available literature with focus on the effect of dust in plasmas (adding the susceptibility of dust particles in Shukla and Melandsø, 1997; Shukla et al., 2001, and others). In the following we follow some of the lines in these papers in view of application to space plasma conditions and with the intention of checking the chances for classical electron "pairing" and possible related effects like formation of macro-electron compounds. Being aware that there is no quantum pairing, we nevertheless keep the expression "pairing" in the following for the reason that we consider attractive forces between two electrons. This is trivially generalised later to account for formation of coagulations of electrons.[3]

[3]An apparently strong argument against the effectiveness of attractive potentials is based on the comparison of the strengths of the attractive and the wave potentials. Attractive potentials are weak and thus apparently negligible, in particular when the wave spectrum is unstably excited. This argument fails for two reasons. First, at the location of the attractive potential the wave potential and the resulting electric force act both on all electrons in the same direction; in contrast the attractive potential acts between electrons, completely independent of the direction of the wave electric field. It cannot be compensated by the wave field. Second, the scale of the wave electric field is given by $k\lambda_{\rm De} \ll 1$, that is, it is large against the Debye length (Fig. 1). In contrast, the attractive potentials between electrons, as shown below, act on scales $\gtrsim \lambda_{\rm De}$, only slightly larger than the Debye length, a scale on which the wave potential is practically constant for all waves under consideration, and no force exists which could compensate for the attractive force even then, when the wave is strongly excited, reaches very high amplitudes and evolves nonlinearly. Its nonlinear evolution being taken care already in the general response function $\epsilon[\omega, k, \Phi(k)]$.

2.1 Ion-acoustic "pairing" potential

Our first example is the response of the test charge potential to the presence of a spectrum of ion-acoustic waves in a plasma, a problem originally treated cursorily by Nambu and Akama (1985). In this case the linear response function is well known. Neglecting damping, its real part is given by

$$\epsilon_{\mathrm{ia}}(\omega, k) = 1 + \frac{1}{k^2\lambda_{\mathrm{De}}^2} - \frac{\omega_i^2}{\omega^2}\left(1 + 3k^2\lambda_{\mathrm{Di}}^2\right), \tag{5}$$

where in the round brackets we iterated the frequency by approximating it with the ion plasma frequency ω_{i}, which produced the ion Debye length λ_{Di}. Putting $\epsilon_{\mathrm{ia}} = 0$ yields the ion-acoustic dispersion relation

$$\omega_{\mathrm{ia}}^2(k) = \frac{\omega_{\mathrm{i}}^2}{1 + 1/k^2\lambda_{\mathrm{De}}^2}\left[1 + \frac{3T_{\mathrm{i}}}{T_{\mathrm{e}}}\left(1 + k^2\lambda_{\mathrm{De}}^2\right) + \frac{k^2\lambda_{\mathrm{De}}^2}{1 + k^2\lambda_{\mathrm{De}}^2}\frac{\delta N}{N}\right] \tag{6}$$

The last term in the bracket on the right results from a possible nonlinear density modulation δN. It vanishes in the long-wavelength regime $k^2\lambda_{\mathrm{De}}^2 \ll 1$. In order to proceed, we need a treatable form of $\epsilon_{\mathrm{ia}}^{-1}$. It is not difficult to show that this can conveniently be written as

$$\frac{1}{\epsilon_{\mathrm{ia}}(\omega, k)} = \frac{k^2\lambda_{\mathrm{De}}^2}{1 + k^2\lambda_{\mathrm{De}}^2}\left(1 + \frac{\omega_{\mathrm{ia}}^2(k)}{\omega^2 - \omega_{\mathrm{ia}}^2(k)}\right), \tag{7}$$

which separates it into two parts. The first term is independent of the presence of ion-acoustic waves. It is thus completely spherically symmetric resulting in the known Debye-screening potential field of the point charge (treated in Neufeld and Ritchie, 1955). Its contribution to the potential at distances $r \gg \lambda_{\mathrm{De}}$, large with respect to the Debye radius, is exponentially small. The nonlinear term in the wave dispersion relation is of higher order and can be neglected meaning that the nonlinear modulation of the wave spectrum is of too large scale for causing a first-order effect in the potential disturbance.

The dominant effect of the test electron interaction with the ion-acoustic wave spectrum is contained in the wave-mediated part. Its main contribution comes from the resonant denominator in frequency space the contribution of which dominates over that of the exponentially decreasing screened repulsing Coulomb potential Φ_{D} outside the Debye sphere.

Equation (7) inserted into the general expression for the test particle potential Eq. (3) yields for the wave-induced contribution

$$\Phi_{\mathrm{ia}}(x, t) = \frac{e\lambda_{\mathrm{De}}^2}{16\pi^2\epsilon_0}\int \frac{\mathrm{d}k\mathrm{d}\omega\ \omega_{\mathrm{ia}}(k)\mathrm{e}^{ik\cdot r}}{1 + k^2\lambda_{\mathrm{De}}^2} \times \left[\frac{\delta(\omega - k\cdot v)}{\omega - \omega_{\mathrm{ia}}(k)} - \frac{\delta(\omega - k\cdot v)}{\omega + \omega_P \mathrm{ia(k)}}\right]. \tag{8}$$

The argument of the δ function depends on the direction of electron velocity $v = vz$, which we arbitrarily chose in z direction. It is then appropriate to treat the integral in cylindrical rather than spherical coordinates with wave number k_z parallel to the electron velocity v and $k_\perp$ perpendicular to it. With ρ being the radius in the plane perpendicular to v, the argument of the exponential becomes $ik\cdot r = ik_z(z - vt) + ik_\perp\rho\sin\phi$. Referring to the definition of Bessel functions, the integration with respect to the azimuthal angle ϕ results in the Bessel function of zero order, and the expression for the potential reads

$$\Phi_{\mathrm{ia}}(z, \rho, t) = \frac{e\lambda_{\mathrm{De}}^2}{16\pi\epsilon_0}\int \frac{\mathrm{d}k_z\mathrm{d}k_\perp^2\, J_0(k_\perp\rho)\,\omega_{\mathrm{ia}}(k_z, k_\perp)}{1 + k_z^2\lambda_{\mathrm{De}}^2 + k_\perp^2\lambda_{\mathrm{De}}^2} \times \left[\frac{\delta(\omega - k_z v)}{\omega - \omega_{\mathrm{ia}}(k)} - \frac{\delta(\omega - k_z v)}{\omega + \omega_{\mathrm{ia}}(k)}\right]\mathrm{e}^{ik_z(z - vt)}\mathrm{d}\omega. \tag{9}$$

Φ_{ia} offers a possible change in sign which opens up the possibility for the potential of becoming attractive for another electron, in which case two electrons may form pairs.

One first makes use of the δ functions to replace $k_z = \omega/v$ in the exponential and elsewhere by performing the k_z integration. The two singularities at $\omega = \pm\omega_{\mathrm{ia}}$ require performing the ω integration in the complex $\omega = \Re(\omega) + i\Im(\omega)$ plane via the principal values of the two integrals and the two residua with integration contour now closed in the lower-half plane, that is, for $z - vt < 0$ and damped ion-acoustic waves $\Im(\omega) < 0$. One readily shows that the principal value vanishes, for each integral contributes $\lim_{\epsilon\to 0}(\ln\epsilon - \ln\epsilon) + i\pi = i\pi$ which cancel when subtracted. The residua yield the resonant result

$$\Phi_{\mathrm{ia}}(z, \rho, t) = \frac{e\lambda_{\mathrm{De}}^2}{8v\epsilon_0}\int \frac{\mathrm{d}k_\perp^2\, J_0(k_\perp\rho)\,\omega_{\mathrm{ia}}(v, k_\perp)}{1 + \omega_{\mathrm{ia}}^2\lambda_{\mathrm{De}}^2/v^2 + k_\perp^2\lambda_{\mathrm{De}}^2} \times \sin\left[\omega_{\mathrm{ia}}(v, k_\perp)\left(\frac{z}{v} - t\right)\right] \tag{10}$$

for the wave–particle interaction part of the electrostatic potential. In this expression the ion-acoustic frequency is implicitly defined through the ion-acoustic dispersion relation. Replacing $k_z = \omega_{\mathrm{ia}}/v$, the latter can be iterated, yielding to lowest order in the long-wavelength regime $k_\perp\lambda_{\mathrm{De}} \ll 1$ that $\omega_{\mathrm{ia}}^2(v, k_\perp) \approx \omega_{\mathrm{i}}^2\lambda_{\mathrm{De}}^2k_\perp^2[1 + \omega_{\mathrm{ia}}^2/k_\perp^2v^2)] \approx c_{\mathrm{ia}}^2k_\perp^2/[1 - (m_{\mathrm{e}}/m_{\mathrm{i}})T_{\mathrm{e}}/K_{\mathrm{test}}]$. Here $K_{\mathrm{test}} = m_{\mathrm{e}}v^2/2$ is the test particle kinetic energy. This becomes simply $\omega_{\mathrm{ia}}^2(v, k_\perp) \approx k_\perp^2c_{\mathrm{ia}}^2/(1 - c_{\mathrm{ia}}^2/v^2)$ with $c_{\mathrm{ia}} \approx \omega_{\mathrm{i}}\lambda_{\mathrm{De}}$ the ion sound velocity.

The wave-number integral must be truncated at the Debye radius $k_\perp\lambda_{\mathrm{De}} \leq 1$ because inside the Debye sphere the point charge potential dominates. This accounts for long wavelengths only. Then the integral becomes

$$\Phi_{\mathrm{ia}}(z, \rho, t) \approx C\int_0^1 \frac{\mathrm{d}\xi\,\xi^2\, J_0(\xi\bar{\rho})}{1 + \xi^2[1 + 1/(v^2/c_{\mathrm{ia}}^2 - 1)]}\ \sin(\beta\xi) \tag{11}$$

with $\xi = k_\perp/\lambda_{\rm De}$, $\bar\rho = \rho/\lambda_{\rm De}$, and $\beta = \zeta(c_{\rm ia}/v)(1 - c_{\rm ia}^2/v^2)^{-1/2}$, $\zeta = (z - vt)/\lambda_{\rm De}$. The constant is $C = (e/4\epsilon_0\lambda_{\rm De})(c_{\rm ia}/v)(1 - c_{\rm ia}^2/v^2)^{-1/2}$. Strictly speaking, this integral with respect to ξ is the sum of its principal value and the contribution of the poles at $\xi_\pm = \pm i(1 - c_{\rm ia}^2/v^2)^{1/2}$. At sufficiently large particle speeds the pole contribution is negligible, and only the principal value counts. This is seen as follows. For resonant particles $v \gtrsim c_{\rm ia}$ the poles are purely imaginary. Extending the singular integral over the entire domain implies that only the positive pole contributes, which is obvious already from Eq. (8) since the exponential vanishes at large r for positive imaginary part of $k_\perp\lambda_{\rm De} = \xi$ only. In performing the path integration the pole is surrounded in negative direction. Taking the residuum yields a term

$$2\pi i|\xi_+|^2 J_0\Big(i|\xi_+|\bar\rho\Big)\ \sin\ \Big(i\beta|\xi_+|\Big) = -2\pi|\xi_+|^2 I_0\Big(|\xi_+|\bar\rho\Big)\sinh\Big(\beta|\xi_+|\Big). \tag{12}$$

$I_0(x)$ is the zero-order modified Bessel function. It is obvious that this entire term for particles close to resonance with $v \gtrsim c_{\rm ia}$, $|\xi_+| \sim O(v^2 - c_{\rm ia}^2)$ is very small, confirming that it can safely be neglected.

When calculating the principal part of the integral, we consider the case $k_\perp\rho < 1$, that is, radial distances perpendicular to the particle velocity less than the ion-acoustic wavelength but large with respect to the Debye length. Shortest distances are thus $\bar\rho = 1$, yielding $J_0(\xi\bar\rho) = J_0(\xi)$ a function of the integration variable only, varying in the interval $0.77 < J_0(\xi) < 1.0$. Its average is $\langle J_0\rangle \approx 0.85$, which we extract from the integral

$$\Phi_{\rm ia}(z,1,t) \approx C'\int_0^1 \xi^2 {\rm d}\xi\ \sin(\beta\xi) = \frac{C'}{\beta^3}\left[\left(1 - \frac{1}{2}\beta^2\right)\cos\beta + \beta\sin\beta - 1\right] \tag{13}$$

with $C' = C\langle J_0\rangle$. The integral is of the same form as in Nambu and Akama (1985). The requirement $\zeta = z - vt < 0$ implies $\beta < 0$. The potential becomes negative whenever the expression in the brackets is positive. The interesting case is when the test particle moves at velocity $v \gtrsim c_{\rm ia}$ exceeding the wave velocity only slightly. Then $|\beta|$ mod $2\pi > 1$, and the dominant term is $\beta^2\cos\beta$ thus confirming Nambu and Akama (1985) and yielding

$$\Phi_{\rm ia}(z,1,t) \approx \left(\Big|C'/\beta\Big|\right)\cos\beta. \tag{14}$$

The potential is attractive in all regions $\cos\beta < 0$ (i.e. $\beta > \pi/2$). In the moving particle frame $\Delta \equiv (z - vt)/\lambda_{\rm De}$ the potential is attractive behind the particle in its wake in regions $\Delta < 0$ for $v \gtrsim c_{\rm ia}$. Here the value of the potential is

$$\Phi_{\rm ia}(\Delta) \approx -\frac{e\langle J_0\rangle}{4\pi\epsilon_0\lambda_{\rm De}|\Delta|}\left|\cos\left[\frac{c_{\rm ia}\Delta}{(v^2 - c_{\rm ia}^2)^{1/2}}\right]\right|, \tag{15}$$

$$\frac{\pi}{2} \lesssim \frac{c_{\rm ia}\Big|\Delta\Big|}{\sqrt{v^2 - c_{\rm ia}^2}} \lesssim \frac{3\pi}{2}. \tag{16}$$

The effective distance $|z - vt| \sim \lambda_{\rm De}$ over which the potential is attractive is thus given by $\ell_{\rm att}/\lambda_{\rm De} \approx (v^2/c_{\rm ia}^2 - 1)^{1/2} > 2/\pi$, mod 2π, that is, the attraction is strongest just outside the Debye length which implies that two electrons one Debye length apart attract each other. In other words, two Debye spheres mutually overlapping by one Debye radius attract each other. For the resonant particle velocity this condition yields $v_{\rm res} > 1.1\,c_{\rm ia}$.

In order to attract another electron, it is clear that the two electrons must move close to each other within a distance $\Delta > 1$ in the region of negative $\Phi_{\rm ia}$, both being in resonance with the wave at velocities $v \gtrsim c_{\rm ia}$. In this case they can form pairs effectively becoming Bosons of either zero or integer spin. We may note that in a magnetic field B with the electrons moving along the field, the ion-sound wave depends on the propagation angle $\cos\theta = k\cdot B/kB$. In this case we have for the sound speed $c_{\rm ia} \to c_{\rm ia}\cos\theta$, and the potential becomes a sensitive function of θ, maximising along B. Moreover, we can set $\rho = 0$ and $\langle J_0(0)\rangle = 1$ as only the distance z along B comes into play.

This attractive potential has to be compared to the wave potential $\Phi_{\rm w}$ the particles are in resonance with. With the assumption $k\lambda_{\rm De} \ll 1$, we are in the long-wavelength regime with the potential assumed being nearly constant over the range of variability of the attractive potential. Thus the attractive force of the trapping wave potential is small. In negative wave phases it adds to that of the particle by confining low-energy electrons in the potential well. These electrons oscillate at the high trapping frequency with their average speeds in resonance with the wave. Wave trapping, though being different in the average, helps attracting as in the attracting potential only the average trapped speed $\langle v\rangle \approx c_{\rm ia}$ counts. The high jitter speed at trapping frequency of the electrons averages out.

Wave-trapped electrons are the best candidates for forming pairs. Moreover, since a pair of charge $2e$ that has been formed in the negative wave potential may well by the same mechanism produce a negative pair potential $\Phi_{\rm pair} = \frac{2}{3}\Phi_{\rm ia}$ over the distance of $3\lambda_{\rm De}$, it may attract other electrons or pairs to form larger macro-particles of large mass and charge but constant mass-to-charge ratio. In the extreme (though possibly unrealistic) case, the maximum number of coagulated electrons could about equal the number of electrons trapped in the wave potential well, since all the negative potentials of the particles involved in producing attractive potentials add to the wave potential. In effect this mechanism may produce macro-electrons of large mass and charge which behave like a single particle and have such properties as exploited in small mass-ratio numerical PIC simulations.

2.2 Electron-acoustic "pairing" potential

Another wave of similar dispersion is the electron-acoustic wave. It is excited wherever the plasma contains two electron populations of different temperatures and densities. Its response function resembles that of ion-acoustic waves with the only difference that two populations of electrons are involved, and ions are assumed forming a fixed charge neutralising background such that for the densities $N_\mathrm{i} = N_\mathrm{c} + N_\mathrm{h}$ where indices c, h refer to the cold and hot electron components. Electron-acoustic waves are high-frequency waves in the sense that $kv_\mathrm{h}, kv_\mathrm{c} \ll |\omega - k\cdot u)_\mathrm{c}|$, where $v_\mathrm{c}, v_\mathrm{h}$ are the thermal speeds of the different electron components. The electron-acoustic dielectric response function in its simplest form reads

$$\epsilon_\mathrm{ea}(k,\omega) = 1 + \frac{1}{k^2\lambda_\mathrm{Dh}^2} - \frac{\omega_\mathrm{c}^2}{(\omega - k\cdot u_\mathrm{c})^2}. \tag{17}$$

The Debye radius for sufficiently large temperature differences $T_\mathrm{h} > T_\mathrm{c}$ is completely determined by the hot component, and for fixed ions there is no need to include the in term. $\mathbf{u}_\mathrm{c}$ is the bulk streaming velocity of cold electrons. The inverse of the dielectric function can again been brought into the same form as for ion-acoustic waves

$$\frac{1}{\epsilon_\mathrm{ea}(k,\omega)} \approx \frac{k^2\lambda_\mathrm{h}^2}{1+k^2\lambda_\mathrm{h}^2}\left(1 + \frac{\omega_\mathrm{ea}^2}{(\omega - k\cdot u_\mathrm{c})^2 - \omega_\mathrm{ea}^2}\right). \tag{18}$$

This is exactly the same form as for ion-acoustic waves, however, now with the electron-acoustic dispersion relation $\omega_\mathrm{ea}^2 = k^2c_\mathrm{ea}^2/(1+k^2\lambda_\mathrm{h}^2)$ and $c_\mathrm{ea}^2 = v_\mathrm{h}^2(N_\mathrm{c}/N_\mathrm{h})$. For this reason, the analysis is the same as for the ion-acoustic wave. The result has already been given by Shukla and Melandsø (1997) and is listed here for completeness only:

$$\Phi_\mathrm{ea} \propto \Big(e/|z-ut|\Big)\cos\Big[|z-vt|/\lambda_\mathrm{h}(1-c_\mathrm{ea}^2/v^2)^{1/2}\Big]. \tag{19}$$

The bulk speed of the electrons has been suppressed here. As before, there are some ranges in which the wave potential at the test charge can be negative and thus attract other electrons. This will, however, only happen in a plasma where two widely separated in temperature electron populations exist of which the colder one is streaming. Interestingly this might be the case in conditions when Bernstein–Green–Kruskal (BGK) electron hole modes are excited. In this case the hole generates a dilute hot electron component which is traversed by a rather cold component of beam electrons. Possibly in this case mutually attracting electrons become possible. Unfortunately, electron-acoustic waves have not been detected in these cases, however, in numerical simulations of electron hole formation. As electron-acoustic waves require strong forcing in order to overcome damping, electron-acoustic waves are not a primary candidate for generating attractive wave potentials.

2.3 Lower-hybrid "pairing" potential

A most important medium frequency wave is the lower-hybrid mode (Huba et al., 1977; Yoon et al., 2002). It propagates in a plasma under almost all conditions on scales below the ion cyclotron radius and frequency. Hence the ions behave non-magnetically while the electrons are completely magnetised being tied to the magnetic field and drifting in the electric field of the wave mode. Lower-hybrid waves can be excited by density gradients, diamagnetic drifts and all kinds of transverse currents $\mathbf{J}_\perp = \sum_\mathrm{s} q_\mathrm{s}N_\mathrm{s}u_{\mathrm{s}\perp}$ in a plasma, where $u_{\mathrm{s}\perp}$ is the perpendicular drift velocity of species s. They are primarily electrostatic, propagating at oblique angle with respect to the magnetic field though being strongly inclined with $k_\| < k_\perp$. Their response function including the test particle Coulomb potential term reads

$$\begin{aligned}\epsilon_\mathrm{lh}(k,\omega) = 1 &+ \frac{1}{k^2\lambda_\mathrm{De}^2} - \frac{\omega_\mathrm{lh}^2}{\omega^2}\left[1 + \frac{m_\mathrm{i}}{m_\mathrm{e}}\frac{k_\|^2}{k_\perp^2}\right. \\ &\left. + \frac{3k^2}{2k_\perp^2}\left(1+\frac{\omega_\mathrm{e}^2}{\omega_\mathrm{ce}^2}\right)k^2\lambda_\mathrm{Di}^2\right], \\ &k_\|/k_\perp \approx \sqrt{m_\mathrm{e}/m_\mathrm{i}}.\end{aligned} \tag{20}$$

The lower-hybrid frequency is defined as $\omega_\mathrm{lh}^2 = \omega_\mathrm{i}^2(1 + \omega_\mathrm{e}^2/\omega_\mathrm{ce}^2)^{-1}$. The term in brackets results from the large argument expansion of the derivative of the plasma dispersion function $Z'(\zeta_\mathrm{i}) = -2[1+\zeta_\mathrm{i}Z(\zeta_\mathrm{i})]$ with $\zeta = \omega/(\omega_\mathrm{i}k\lambda_\mathrm{Di})$ the argument for the immobile ions. This response function is formally of the same structure as the ion-acoustic response function Eq. (5). Thus defining

$$\begin{aligned}\omega_\mathrm{lh}^2(k) = \frac{\omega_\mathrm{lh}^2}{1+1/k^2\lambda_\mathrm{De}^2}&\left[1+\frac{m_\mathrm{i}}{m_\mathrm{e}}\frac{k_\|^2}{k_\perp^2}\right. \\ &\left.+\frac{3k^2}{2k_\perp^2}\left(1+\frac{\omega_\mathrm{e}^2}{\omega_\mathrm{ce}^2}\right)k^2\lambda_\mathrm{Di}^2\right]\end{aligned} \tag{21}$$

the whole formalism developed for ion-acoustic waves can be applied to lower-hybrid waves. We write for the inverse response function

$$\frac{1}{\epsilon_\mathrm{lh}(\omega,k)} = \frac{k^2\lambda_\mathrm{De}^2}{1+k^2\lambda_\mathrm{De}^2}\left(1 + \frac{\omega_\mathrm{lh}^2(k)}{\omega^2 - \omega_\mathrm{lh}^2(k)}\right). \tag{22}$$

Again, the Debye-screening term outside the brackets is of no interest at distances $r > \lambda_\mathrm{De}$. The contribution to the wake potential comes from the integral in Eq. (9) with $\omega_\mathrm{ia}(k)$ replaced by $\omega_\mathrm{lh}(k)$ and $k_z \equiv k_\| = \omega/v\sqrt{\mu}$, where $k_\| \sim k_\perp\sqrt{\mu}$,

$\mu = m_\mathrm{e}/m_\mathrm{i}$, for nearly perpendicular wave propagation. Performing the ω integration reproduces a form similar to Eq. (10)

$$\Phi_\mathrm{lh}(z,\rho,t) = \frac{e\lambda_\mathrm{De}^2}{8v\epsilon_0}\int \frac{\mathrm{d}k_\perp^2\, J_0(k_\perp\rho)\omega_\mathrm{lh}(v,k_\perp)}{1+\lambda_\mathrm{De}^2\omega_\mathrm{lh}^2(v,k_\perp)/\mu v^2 + k_\perp^2\lambda_\mathrm{De}^2} \times \sin\left[\frac{\lambda_\mathrm{De}\,\omega_\mathrm{lh}(v,k_\perp)}{\sqrt{\mu}\,v}\frac{(z-vt)}{\lambda_\mathrm{De}}\right] \tag{23}$$

but now including the more complicated lower-hybrid frequency Eq. (21). We simplify the lower-hybrid frequency by observing $k_\parallel^2/\mu k_\perp^2 \sim 1$. In dense plasma the last term in the brackets becomes $k_\perp^2\lambda_\mathrm{Di}^2(\omega_\mathrm{e}^2/\omega_\mathrm{ce}^2) \sim k_\perp^2 r_\mathrm{ce}^2$ which is of the order of the electron gyroradius-to-wavelength squared, being small for completely magnetised electrons. Hence, $\omega_\mathrm{lh}^2\lambda_\mathrm{De}^2 \sim 2V_\mathrm{A}^2(v_\mathrm{e}/c)^2 \equiv c_\mathrm{lh}^2$ will be used in the factor in front of the sine function. The lower-hybrid wave in this case propagates at the Alfvén speed V_A corrected by the ratio of electron thermal to light velocity. In this approximation and with $\xi = k_\perp\lambda_\mathrm{De}$, we have for the lower-hybrid dispersion relation

$$\lambda_\mathrm{De}^2\omega_\mathrm{lh}^2(v,k) \approx \frac{c_\mathrm{lh}^2\,\xi^2(1+\omega^2\lambda_\mathrm{De}^2/v^2\xi^2)}{1+k^2\lambda_\mathrm{De}^2}\left(1+\frac{k_\parallel^2}{\mu k_\perp^2}\right) \approx \frac{2c_\mathrm{lh}^2\,\xi^2}{1-c_\mathrm{lh}^2/v^2} \tag{24}$$

which is to be used in the above integral in the long-wavelength approximation $k_\perp\lambda_\mathrm{De} \equiv \xi < 1$ and $v \gtrsim c_{\mathrm{lh},\parallel} = c_\mathrm{lh}/\sqrt{\mu} > c_\mathrm{lh}$. The last version on the right results from iterating the frequency $\omega = \omega_\mathrm{lh}(v,k)$. Within these approximations and restricting to the interval $\xi \lesssim 1$ for long wavelengths, the potential becomes

$$\Phi_\mathrm{lh}(z,\bar\rho,t) \approx C_\mathrm{lh}\int \frac{\xi^2\,\mathrm{d}\xi\; J_0(\xi\bar\rho)}{1+\xi^2(1+2c_{\mathrm{lh},\parallel}^2/v^2)}\sin\Big(\beta_\mathrm{lh}\xi\Big) \approx C'_\mathrm{lh}\int_0^{1/\sqrt{3}} \xi^2\mathrm{d}\xi\,\sin(\beta_\mathrm{lh}\xi) \tag{25}$$

$$C_\mathrm{lh} = \frac{C'_\mathrm{lh}}{\langle J_0\rangle} = \frac{e}{2\epsilon_0\lambda_\mathrm{De}}\frac{c_\mathrm{lh}}{(v^2-c_\mathrm{lh}^2)^{1/2}}, \qquad \beta_\mathrm{lh} \equiv \frac{2c_{\mathrm{lh},\parallel}\Delta}{(v^2-c_{\mathrm{lh},\parallel}^2)^{1/2}} < 0, \tag{26}$$

where $\Delta \equiv (z-vt)/\lambda_\mathrm{De}$. One may note that in the only interesting long-wavelength regime the factor multiplying ξ^2 in the denominator is at most 3. In order to neglect the entire term $\xi^2(1+2c_\mathrm{lh}^2/v^2) \ll 1$ and being able to analytically solve the integral one thus requires that the upper limit of the integral is taken as $\xi < 1/\sqrt{3}$. Averaging the Bessel function over this interval again produces the numerical factor $\langle J_0\rangle$. In the argument of the sine function the larger parallel wave velocity $c_{\mathrm{lh},\parallel} = c_\mathrm{lh}/\sqrt{\mu}$ appears. It is due to the higher phase velocities of the lower-hybrid waves parallel rather than perpendicular to the magnetic field, while the test electron moves along the magnetic field at velocity $v \gtrsim c_{\mathrm{lh},\parallel}$ being in resonance with the wave.

With these assumptions the integration of the sine function with respect to $k_\perp$ can be performed as before and an attractive wake potential is obtained under similar conditions as for the ion-acoustic wave Eq. (15):

$$\Phi_\mathrm{lh}(\Delta) \approx \frac{C'_\mathrm{lh}}{3|\beta_\mathrm{lh}|}\cos\frac{\beta_\mathrm{lh}}{\sqrt{3}} \approx \frac{e\langle J_0\rangle}{12\epsilon_0\lambda_\mathrm{De}}\frac{\sqrt{\mu}}{\Big|\Delta\Big|}\left(\frac{v^2-c_{\mathrm{lh},\parallel}^2}{v^2-c_\mathrm{lh}^2}\right)^{\frac{1}{2}} \times\cos\left[\frac{2c_{\mathrm{lh},\parallel}\Big|\Delta\Big|}{\sqrt{3\Big(v^2-c_{\mathrm{lh},\parallel}^2\Big)}}\right]. \tag{27}$$

This potential becomes negative for $\frac{1}{2}\pi < |\beta_\mathrm{lh}|/\sqrt{3} < \frac{3}{2}\pi \mod 2\pi$, in which case it attracts a neighbouring parallel electron. An attractive potential requires $v \gtrsim c_\mathrm{lh}/\sqrt{\mu} \approx 43\,c_\mathrm{lh}$ in an electron–proton plasma. As a consequence the fraction under the square root does not shorten out but becomes small of the order of $o\left(1-c_{\mathrm{lh},\parallel}^2/v^2\right) \sim O\left(\sqrt{\mu}\right)$. Under the condition on the argument of the cos function the amplitude of the potential is of the order of

$$\frac{C'_\mathrm{lh}}{3|\beta_\mathrm{lh}|} \lesssim \frac{e\langle J_0\rangle}{3\sqrt{3}\epsilon_0\lambda_\mathrm{De}}\frac{c_\mathrm{lh}}{v} \tag{28}$$

which is small of the order of the ratio $c_\mathrm{lh}\,/\,v \sim \sqrt{\mu}$. Nevertheless, lower-hybrid waves may attract some resonant electrons in parallel motion along the magnetic field. In the transverse direction any electrons gyrate and thus are insensitive to attraction. Any potential generated will just cause a cross-field electron drift weakly contributing to local current fluctuations.

2.4 Buneman mode-mediated inter-electron potential

A most important plasma wave is the current driven non-magnetic Buneman mode (Buneman, 1958, 1959). It occurs under conditions of collisionless shocks, in collisionless guide field reconnection (Drake et al., 2003; Cattell et al., 2005), and in auroral physics, in all cases producing highly dynamical localised electron structures of the type of BGK modes which trap electrons and cause violent effects in plasma dynamics (Newman et al., 2001). Again accounting for the presence of test electrons, the dielectric response function of the Buneman mode is

$$\epsilon(\omega,k) = 1+\frac{1}{k^2\lambda_\mathrm{De}^2}-\frac{\omega_\mathrm{i}^2}{\omega^2}-\frac{\omega_\mathrm{e}^2}{(\omega-ku)^2}, \tag{29}$$

with u the current drift velocity of the electrons, and k the one-dimensional wave number. For the Buneman mode one has $k \approx \omega_e/u$ and $\omega_i \ll \omega \ll \omega_e$. Under these conditions the (nonlinear) version of the Buneman response function becomes

$$\epsilon_B(\omega,k) = 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{\mu}{2}\left(\frac{\omega_e}{\omega}\right)^3\left(1+\frac{3}{2}\frac{\delta N}{N}\right). \tag{30}$$

The Buneman dispersion relation is obtained as

$$\omega_B^3(k) = -\omega_e^3\,\frac{\mu}{2}\,\frac{k^2\lambda_{De}^2}{1+k^2\lambda_{De}^2}\left(1+\frac{3}{2}\frac{\delta N}{N}\right), \tag{31}$$

where we retained the nonlinear modulation term proportional to the density variation δN. In equilibrium it becomes $\delta N/N = -(\epsilon_0/4m_i c_{ia}^2 N)\,|\delta E_B|^2$ which is proportional to the Buneman electric field intensity causing hole formation. In the following this term will be neglected. We note that the solution $\omega_B(k) = \Re(\omega_B) + i\Im(\omega_B)$ has a non-negligible imaginary part which must be taken into account. Inverting the response function yields,

$$\frac{1}{\epsilon_B(\omega,k)} = \frac{k^2\lambda_{De}^2}{1+k^2\lambda_{De}^2}\left(1+\frac{\omega_B^3(k)}{\omega^3-\omega_B^3(k)}\right). \tag{32}$$

2.4.1 Attractive potential in linear theory

The structure of this function is more complicated than in the ion-acoustic case which is due to the higher power in frequency and its imaginary part. This function is to be used in Eq. (1). Again, the first term just reproduces the Debye screening and can thus be dropped. In order to treat the integral of the second term, we again assume that the electron moves in z direction at velocity v. Rewriting the integral in cylindrical coordinates and replacing $k_\parallel = \omega/v$ as required by the delta function, we find

$$\Phi_B(z,\rho,t) = \frac{e\lambda_{De}^2}{16\pi\epsilon_0}\int \frac{\mathrm{d}\omega \mathrm{d}k_\perp^2\, J_0(k_\perp\rho)}{1+\omega^2\lambda_{De}^2/v^2+k_\perp^2\lambda_{De}^2} \times \frac{\omega_B^3(k_\perp,v)\,\exp[i\omega(z-vt)/v]}{\left[\omega^3-\omega_B^3(k_\perp,v)\right]}. \tag{33}$$

Treating the ω integral is complicated by the third power of the frequency. It requires expansion of the last term into a Laurent series. Since we know that ω_B is a solution of the dispersion relation, the denominator can be expanded around $\omega=\omega_B$ yielding in the denominator $3\omega_B^2(\omega-\omega_B)\left[1+(\omega-\omega_B)/\omega_B+\frac{1}{2}(\omega-\omega_B)^2/\omega_B^2\right]$. The bracket can then be further expanded. Ultimately applying the residuum theorem, only the first term survives producing

$$\Phi_B(z,\rho,t) = \frac{ie\lambda_{De}^2}{4\epsilon_0}\int \frac{k_\perp \mathrm{d}k_\perp}{1+\omega_B^2\lambda_{De}^2/v^2+k_\perp^2\lambda_{De}^2} \times J_0\left(k_\perp\rho\right)\omega_B\left(k_\perp,v\right)\exp\left[i\omega_B(z-vt)/v\right] \tag{34}$$

and we must, for $\Im(\omega)>0$, require that $z-vt>0$ and integrate over the positive frequency half-space. Indeed, solving the dispersion relation still, for completeness, keeping the nonlinear term, we obtain the usual Buneman frequency and growth rate

$$\Re(\omega_B) \approx \frac{\omega_e}{(1+1/k^2\lambda_{De}^2)^{1/3}}\left(\frac{\mu}{16}\right)^{\frac{1}{3}}\left(1+\frac{1}{2}\frac{\delta N}{N}\right), \qquad \Im(\omega_B) = \sqrt{3}\,\Re(\omega_B). \tag{35}$$

Hence, electrons in resonance with the wave lag slightly behind the wave. The integral may be written as a derivative with respect to $\zeta = (z-vt)v_e/v\lambda_{De}$. Further simplifying the denominator and defining $\bar\omega = \omega_e(\mu/16)^{1/3}(1+\delta N/2N) \approx 0.03\omega_e(1+\delta N/2N)$ the integral becomes

$$\Phi_B(z,\bar\rho,t) \approx \frac{e}{4\epsilon_0\lambda_{De}}\partial_\zeta\int_0^1 \xi\mathrm{d}\xi\, J_0(\xi\bar\rho) \times \exp\left[-\bar\omega\xi^{\frac{2}{3}}\left(\sqrt{3}-i\right)\zeta\right]. \tag{36}$$

Changing variables and solving for the integral and restricting to the dominant term, we find that

$$\Phi_B(\zeta,\rho=0) \approx \frac{3}{4}\frac{e}{\epsilon_0\lambda_{De}\bar\omega\zeta}\exp\left(-\zeta\bar\omega\sqrt{3}\right) \times\left[\cos\left(\bar\omega\zeta+\frac{\pi}{6}\right)+i\sin\left(\bar\omega\zeta+\frac{\pi}{6}\right)\right] \tag{37}$$

holding for $\zeta>0$. Only the real part of the potential has physical relevance, the imaginary part causing a spatial undulation along ζ of wavelength $6\bar\omega/11\pi$. We thus find that the potential can indeed become attractive when the cos function is negative, that is, in the interval $\frac{1}{3}\pi \lesssim \bar\omega\zeta \lesssim \frac{4}{3}\pi$ and for resonant electrons lagging slightly behind the wave. This last condition can also be written

$$\frac{\pi}{3} \lesssim 0.03\,\frac{v_e}{v}\frac{|z-vt|}{\lambda_{De}} \lesssim \frac{4\pi}{3}. \tag{38}$$

Such electrons are presumably trapped in the wave potential well which confines them to the interior of holes generated by the Buneman mode. For the distance on which the potential is attractive the last expression yields

$$\bigg|(z-vt)\bigg|_{\mathrm{att}} \gtrsim 10\,\pi\,(v/v_e)\lambda_{De}. \tag{39}$$

For the Buneman mode one requires that $u>v_e$. Electron holes arising from Buneman modes extend up to several $\sim\lambda_{De}$ (Newman et al., 2001). They are thus well capable of allowing trapped slow electrons of velocity in the narrow interval $v_e<v<u$ to experience attracting inter-electron potentials and, in principle, form classical "pairs" or larger compounds.

This attractive potential caused by the Buneman mode is weak. This is obvious from the exponential factor $\exp(-\bar{\omega}\zeta\sqrt{3})$. Inserting for μ and using the condition Eq. (38) with $v_e/v < 1$, it is found that this factor is of the order of $\lesssim 0.007$. Moreover, though the imaginary part of the potential plays no role in the sign of the potential, it implies a periodic modulation of the electric field along z which is obtained when taking the derivative $E_z \propto -\partial\Phi_B/\partial\zeta$. This modulation is, however, spatially damped away by the exponential factor. Hence the potential becomes indeed weakly attractive only in the near zone given by Eq. (38).

2.4.2 Undamped contribution of the singularity in ξ

For completeness we check for the resonant contribution of the ξ integral. This is complicated by the wave-number dependence of the Buneman dispersion relation induced by the presence of the test charge. Iteratively, the remaining k dependence of the Buneman dispersion relation is reduced to $k_\perp$ only. With this in mind, the denominator in Eq. (34) put to zero becomes

$$\xi^2 + \alpha_B \xi^{\frac{4}{3}} + 1 = 0 \tag{40}$$

where $\alpha_B = (\mu/16)^{2/3}(1 + i\sqrt{3})$. Defining $\bar{\xi} = \xi^{2/3}$, this becomes a third-order equation $\bar{\xi}^3 + \alpha_B\bar{\xi}^2 + 1 = 0$ the solution of which is complicated by the complexity of the coefficient α_B. In general it has one real and two complex solutions. The real solution is of no interest as it only contributes to a weak deformation of the Debye sphere. In order to obtain the complex solutions, we may refer to the smallness of $\xi \ll 1$ in the long-wavelength regime and neglect the third-order term. Solving for $\bar{\xi}$ yields four solutions

$$\xi_{1,\ldots,4} \approx \pm\left(\pm\sqrt{\alpha_B^{-3}}\right)^{\frac{1}{2}} = \pm\begin{pmatrix} 1 \\ i \end{pmatrix} a^{-1}\mathrm{e}^{-i\pi/4} \tag{41}$$

with $a \equiv (\mu/16)^{1/2}$. Checking with these solutions for the exponential in Eq. (34) it can be shown that of the solutions in the upper row only the solution ξ_1 with the $+$ sign converges. Its pole lies in the lower-half plane. The pole of the converging lower-row solution ξ_3 lies in the upper half plane and corresponds also to the $+$ sign. The denominator of the integral can thus be written $(\xi^2 - \xi_{1,2}^2)(\xi^2 - \xi_{3,4}^2)$ where only the solutions ξ_1, ξ_3 contribute. The integrand splits into the two resonant terms

$$\frac{\xi_{1,2}^2 - \xi_{3,4}^2}{\alpha_B\xi^{4/3} + 1} = \left[\frac{1}{(\xi - \xi_1)(\xi - \xi_2)} - \frac{1}{(\xi - \xi_3)(\xi - \xi_4)}\right]. \tag{42}$$

The first term on the right contributes a factor $-2\pi i$, the second a factor $2\pi i$ which, when including the minus sign in the bracket, yields a common factor $-2\pi i$. Moreover, $\xi_{1,2}^2 - \xi_{3,4}^2 = -2i/a^2$. Hence a factor $-2\pi i a^2/-2i = \pi a^2$ results. Since $\xi_2 = -\xi_1, \xi_4 = -\xi_3$, a further factor $a/2$ appears which makes a final common factor $\pi a^3/2$. In addition, the two singular terms are multiplied: the first by $\mathrm{e}^{i\pi/4}$ and the second by $\mathrm{e}^{-i\pi/4}$.

Solving for the residues at small $\rho \approx 0$, one again obtains a complex potential[4] which, after some simple but lengthy algebra, yields for the real and imaginary parts of the singular integral contribution to the potential

$$\begin{aligned}
\Re\Phi_B^{sg}(\zeta) \approx{}& \\
& A\cos\frac{\zeta}{2}\left[\sinh\frac{\zeta}{\sqrt{3}}\cos\frac{\pi}{12} - \cosh\frac{\zeta}{\sqrt{3}}\sin\frac{\pi}{12}\right] \\
& -\sin\frac{\zeta}{2}\left[\sinh\frac{\zeta}{\sqrt{3}}\cos\frac{\pi}{12} + \cosh\frac{\zeta}{\sqrt{3}}\sin\frac{\pi}{12}\right], \\
\Im\Phi_B^{sg}(\zeta) \approx{}& \\
& A\cos\frac{\zeta}{2}\left[\sinh\frac{\zeta}{\sqrt{3}}\cos\frac{\pi}{12} + \cosh\frac{\zeta}{\sqrt{3}}\sin\frac{\pi}{12}\right] \\
& -\sin\frac{\zeta}{2}\left[\sinh\frac{\zeta}{\sqrt{3}}\cos\frac{\pi}{12} - \cosh\frac{\zeta}{\sqrt{3}}\sin\frac{\pi}{12}\right],
\end{aligned} \tag{43}$$

where $A \equiv e\pi a^2/4\epsilon_0\sqrt{2}\lambda_{De}$. For ζ positive and small, $0 \lesssim \zeta < 1$, that is, in the domain of largest interest, the dominant term of the real part becomes

$$\Re\Phi_B^{sg}(\zeta) \approx -A\sin(\pi/12)\cos\left(\frac{1}{2}\zeta\right)\cosh\left(\zeta/\sqrt{3}\right) \tag{44}$$

This contribution to the electrostatic potential is both attractive and not exponentially damped. It thus represents an important, in fact the dominant, contribution to the attractive electric force exerted by Buneman modes. In contrast to ion-acoustic wave mediated potentials, the singularity of the ξ integral in presence of the Buneman mode therefore adds substantially to the attractive "pairing" potential in the near zone $\zeta \gtrsim 0$ which acts on the slow electron component and causes electron coagulation possibly leading to the formation of electron compounds or macro-electrons in Buneman turbulence. As before, the imaginary part of the potential contribution merely causes a spatial undulation of the potential.

2.4.3 Weakly nonlinear Buneman mode

The Buneman mode is a strong wave in the sense that it grows very fast, actually close to explosive growth. This has a profound effect on the plasma which appears as hole formation, with $\delta N \neq 0$ reacting on the wave. In a simplified theory this reaction is most easily described by taking the variation of the Buneman frequency $\delta\omega = \delta\Re(\omega_B)$ with respect to both density and wave number (Treumann and Baumjohann, 1997). The latter is varied with respect to $k_B = \omega_e/u$, yielding

$$\delta\omega \approx \Re(\omega_B)\left[\frac{1}{3}\left(\frac{u}{v_e}\right)^2 k^2\lambda_e^2 + \frac{1}{2}\frac{\delta N}{N}\right], \qquad \begin{array}{l} k < k_B, \\ v_e < u \end{array} \tag{45}$$

[4]The case $\rho \neq 0$ produces a series of Bessel functions of complex argument which just provides another severe mathematical complication without adding to any further physical insight.

It is customarily interpreted as an operator equation acting on the Buneman mode electric field envelope $E(z,t)$. This procedure results in a nonlinear Schrödinger equation[5]

$$\left[i\frac{\partial}{\partial\tau} + \frac{1}{2}\nabla^2_{\bar z} + \eta \left|E(\bar z,\tau)\right|^2 \right] E(\bar z,\tau) = 0 \tag{46}$$

where $\tau = \Re(\omega_B)t$, $\bar z = \sqrt{6}\omega_e z/u$. The coefficient $\eta = \epsilon_0/8m_i c_{ia}^2 N$ of the nonlinear term results from the density response of the plasma to the presence of the finite amplitude Buneman wave.

The stationary solution in the comoving frame of the Buneman wave is, in this approximation, a caviton of amplitude $E(\bar z) = E_m/\cosh(\bar z/L)$ of width $L = 1/\left(E_m\bar\eta^{1/2}\right)$ and maximum dip amplitude E_m. In this comoving frame $\bar\eta = \epsilon_0/8m_i(c_{ia}-u)^2N \approx \epsilon_0/8m_iu^2N$ for $u \gg c_{ia}$. Electrons trapped in the cavitons have velocities

$$v < (\epsilon_0/m_eN)^{\frac{1}{2}}E_m. \tag{47}$$

Oscillating back and forth in the caviton, electrons in their backward traveling phase of motion or near their turning points at the boundaries of the cavitons are sensitive to attraction. Hole-passing electrons in either direction, on the other hand, are not in resonance and thus do not experience any attraction.

2.4.4 Strong nonlinearity: electron hole effect

These arguments hold for weakly modulated Buneman modes. As noted above, the Buneman mode is, however, a strong wave which during its evolution causes electron holes to evolve from Bernstein–Green–Kruskal (BGK) modes which cannot be described by the above approximate weakly nonlinear theory. In this case the variation of the density

$$|\delta N/N| \lesssim 1 \tag{48}$$

becomes itself of the order of the density.

Under this condition one may assume that in the Buneman dispersion relation Eq. (35)

$$\delta N/N \approx -\eta|E(\bar z,\tau)|^2 \tag{49}$$

in which case the effective plasma frequency

$$\bar\omega = \omega_e\left(1-\left|\frac{\delta N}{N}\right|\right)^{\frac{1}{2}}\left(\frac{\mu}{16}\right)^{\frac{1}{3}} \ll \omega_e\left(\frac{\mu}{16}\right)^{\frac{1}{3}} \tag{50}$$

becomes very small, yielding that $\bar\omega \approx 0$ in the exponential damping factor in electron holes vanishes – this is a very important fact.

The attractive potential under the condition of electron hole generation becomes undamped, and the condition Eq. (38) assumes full validity. This is the case when the Buneman mode evolves into BGK-mode electron holes as observed in several places in space, the aurora and strong collisionless reconnection. It then becomes capable of contributing to the proposed classical "pairing" or coagulation of electrons inside an electron hole affecting the low-velocity trapped-electron component. As before, passing electrons are immune to any attractive potentials and coagulation.

2.5 Summary

In this paper we examined four types of plasma waves for their capability of causing attraction between two electrons in close distance. All four wave families can, under certain conditions, contribute. Attraction is a purely classical effect which just resembles real quantum pairing of electrons in electron–phonon interaction at low temperatures in solid state physics. Nevertheless the mechanisms are similar in the sense that they imply electron-wave interaction. This lets one ask whether the multiple classical "pairing" (coagulation) [6] may have observable effects. In the last section we present a few speculative hypotheses in this direction.

3 Discussion and conclusions: possible effects

Of all the plasma waves checked, the most promising candidates for "pairing" are ion-acoustic waves. These had been proposed already by Nambu and Akama (1985) in view of application in non-magnetised dusty plasma. Such waves populate the solar wind and magnetosheath where they might produce attractive potentials and generate a minor component of heavy cold coagulated electrons. Electron-acoustic waves, because of their very strong damping, are no really good promising candidate. Lower-hybrid waves propagating into a nearly perpendicular direction have weak parallel potentials only, though we have given arguments for attractive potentials generated by them as well. Large amplitude linear Buneman modes, a particularly important wave mode, suffer from exponential damping.

However, Buneman modes when evolving into electron holes from BGK modes, the density modulation becomes large and – as argued above – the exponential damping factor

[5] Strictly speaking, for the strongly growing Buneman mode one should also account for the variation of the imaginary frequency (growth rate). This results in a complex nonlinear Schrödinger equation $\left[i\partial_\tau + \frac{1}{2}\nabla^2_{\bar z} + \left(1+i\sqrt{3}\right)\eta \left|E(\bar z,\tau)\right|^2 \right] E(\bar z,\tau) = 0.$ Equations of this kind are known from Landau–Ginzburg theory in many-particle quantum statistics but have not yet been considered in plasma physics.

[6] Another term in place of multiple classical "pairing" or coagulation would be "bunching". However, bunching has the connotation of particles being bunched into a common dynamical phase, for instance in their gyrational motion in an external magnetic field as used in free-electron laser and electron-cyclotron maser theory. Since coagulation meant in this paper is a different process, we prefer avoiding use of this term.

is strongly reduced. In view of applications, this is the most interesting case involving Buneman modes for it causes susceptible attractive potentials evolving in the interior of an electron hole. Our calculations do, however, apply only to single wave modes trapped inside a hole. In order to account for the effect of the modulated wave spectrum it would be necessary to integrate over the hole-trapped wave spectrum under the restrictive condition imposed by the resonance condition limited to the necessary condition for producing attractive potentials. The latter two separate out just a small group of particular resonant electrons from the trapped electron component for each of the wave numbers k in the spectrum of hole-trapped waves. Electrons at the bottom of the hole potential are clearly not involved in the resonance and attraction; they are at rest. This all implies that the number of resonant electrons ready for attraction will be very small. It consists of the fraction, say $\alpha_{\rm res} \ll 1$ of resonant particles satisfying the attractive condition cut out of the trapped electron distribution located in a shell of (negative) attractive potential just outside the Debye sphere being of spatial extension $r \sim \nu\lambda_{\rm D}$ with $\nu < 1$. For a trapped electron density N the fraction of electrons per Debye sphere in this narrow shell is $\sim \nu N$. Of these just a fraction $\alpha_{\rm res}$ is in resonance. This yields per Debye sphere a fraction of $\sim \nu\alpha_{\rm res}N \ll N$ available for compound formation. Clearly this fraction is very small.

In principle, one could also think of electromagnetic plasma waves causing attractive potentials. The candidates would, however, only be electromagnetic waves possessing sufficiently large magnetic field aligned electric fields. Naturally, low-frequency electromagnetic waves have relativistically small electric components. Hence, the only candidates could be highly oblique whistlers, which generally resemble lower-hybrid modes and need not be discussed further, kinetic Alfvén waves which are known to possess large-scale and comparably strong electric fields, in particular in the auroral region, but also on the ion-inertial scale near reconnection sites, and the extraordinary electromagnetic mode. Of these, only kinetic Alfvén waves are worth being checked. This will be reserved for a separate investigation. It requires an electromagnetic treatment involving the magnetic vector potential.

3.1 Mass and charge of prospective coagulations

In solid state physics, electron pair formation is related to super-fluid and super-conducting behaviour of matter (Fetter and Walecka, 1971; Huang, 1987; Ketterson and Song, 1999) in metals and semi-conductors which are based on the fact that pairing electrons become Bosons with either vanishing or integer spin. At low temperatures they are capable of releasing their kinetic energy until condensing in their lowest energy level which, in a magnetic field, is the lowest Landau level $\frac{1}{2}\hbar\omega_{\rm ce}$ (Landau, 1930).

Classical "pairing" produces compounds of electrons which attract each other. Each electron may become surrounded by other weakly bound electrons. This happens on the scale of the Debye length (Fig. 1). Such compounds have large masses and charges

$$m_* = n_{\rm com}m_{\rm e}, \quad q_* = -n_{\rm com}e \tag{51}$$

with $n_{\rm com}$ the number of electrons in the compound, but constant charge-to-mass ratio $e\,/\,m_{\rm e}$. The mass increase affects thermal speed, momentum and kinetic energy. The charge will be compensated by the unchanged number of ions.

It remains an open question whether or not classical "pairing" or coagulation will actually take place. As noted, the presence of an attractive potential which is responsible for the attractive force between neighbouring electrons, is just the necessary condition for subsequent coagulation of electrons to form classical pairs or larger electron compounds. Real compound production requires, in addition, the observation of the sufficient conditions. These are more complicated to investigate than the mere though already quite involved generation of attractive potentials given in this paper.

The necessary (attractive potential) condition consists of two parts. In brief, for an electron experiencing the attractive potential force these are the resonance condition imposed on the electron and the requirement that the electron is localised at the right location in space where the potential is attractive. The former depends on the wave mode. The latter, as has been noted, says that for becoming attracted a resonant electron must be located at a radial distance from the attracting electron outside but very close to the latter's Debye sphere. Inside the Debye sphere the potential is repulsive. At distance larger than the Debye sphere the attractive force rapidly decays with distance. Attraction is, hence, limited to a thin shell of some thickness d located at radial distance $\lambda_{\rm De}$ from the attracting electron. Solving the sufficient conditions not only requires determining the attractive force (taking the radial gradient of the attractive potential) but also integrating in momentum space over the resonant particle distribution in presence of a given wave spectrum, and integrating spatially over the attractive shell.

Such a calculation can only be done numerically and remains to be a formidable task. Still, it does not yet provide information about the (average) number $n_{\rm com}$ of particles in a single compound. This number depends on how many Debye spheres become correlated in the attraction process, a number which is not known a priori.

3.2 Electron cooling

Since only a small number of electrons participate in attraction, their distribution function is just a narrow cut out of the distribution of all electrons available in the volume. Figure 2 sketches the situation for the case of ion-acoustic waves which may originally have been unstably excited in a thermally imbalanced ion–electron plasma $T_{\rm e} > T_{\rm i}$ as shown by

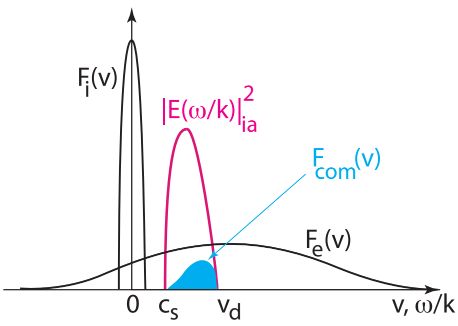

Figure 2. Phase space of ion-acoustic waves excited by the ion-acoustic instability. Shown are the one-dimensional background ion $F_\mathrm{i}(v)$ and electron $F_\mathrm{e}(v)$ distributions. Ion-acoustic wave with spectrum $|E(\omega/k)|^2_\mathrm{ia}$ evolve at phase velocities above the minimum of c_ia in the range $c_\mathrm{s} < \omega/k < v_\mathrm{d}$. The electron pair distribution function $F_\mathrm{com}(v)$ produced in the high-phase speed range is shown schematically in blue. One may note the very low velocity spread of the pair distribution indicating the much lower pair than original electron background temperature $T_\mathrm{pair} \ll T_\mathrm{e}$.

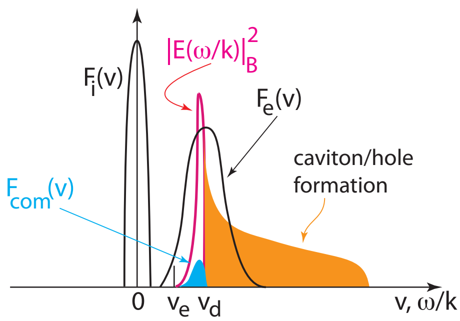

Figure 3. Phase space of Buneman modes excited with spectrum $|E(\omega/k)|^2_b$ and wave number $k = \omega_\mathrm{e}/v_\mathrm{d}$ evolving at phase velocities above v_e for $u \equiv v_\mathrm{d} > v_\mathrm{e}$. The spectrum is very narrow in phase velocity. The electron pair distribution function $F_\mathrm{com}(v)$ produced (blue) has similar width as the spectrum and is thus much colder than the original electron distribution. In caviton formation the spectrum extends to much larger phase velocities which, however, has no remarkable effect on the pair distribution.

the two distributions $F_\mathrm{i}(v), F_\mathrm{e}(v)$ in one-dimensional phase space. This is the canonical case of ion-acoustic wave excitation. The ion-acoustic wave spectrum exists in a narrow phase velocity range $c_\mathrm{s} < \omega_\mathrm{ia}/k < v_d$ as shown in red. c_s is the minimum of the ion-acoustic wave phase velocity. Attractive potentials can be generated only at finite wave amplitudes and for electron velocities $v \gtrsim c_\mathrm{ia}$. The resulting low-density pair distribution is shown in blue.

One may note the very narrow velocity spread of the attracted distribution $F_\mathrm{com}(v)$ which is at most as wide as the ion-acoustic wave spectrum corresponding to a rather low-temperature $T_\mathrm{com} \ll T_\mathrm{e}$ of the electrons participating in attraction and available for compound formation. Their maximum speed is sufficiently far below v_d. Compound distributions are cold.

Figure 3 is for the Buneman case which holds for $u \gtrsim v_\mathrm{e}$. The excited spectrum in this case is as well extremely narrow with phase velocity spread of the same order as the linearly excited Buneman waves, that is, $\Delta v \sim u - v_\mathrm{e}$. Buneman modes are excited for $u \gtrsim v_\mathrm{e}$ just above the electron thermal speed. One may note that the reactively growing wave readily reduces any speed $u \gg v_\mathrm{e}$ to values marginally exceeding v_e. Consequently, the compound distribution which is at most as wide as the Buneman spectrum, also has low-temperature $T_\mathrm{com} \lesssim m_\mathrm{e}|u - v_\mathrm{e}|^2 \ll T_\mathrm{e}$.

Buneman modes are known to evolve into electron holes. In this case the hole-trapped electrons become heated in the trapped wave spectrum. Clearly, the prospectively attracted electrons or compounds formed will, in the long term, participate in this heating. However, formation of attractive potentials and attraction are almost immediate processes in the interaction of resonant electrons with one of the propagating waves trapped in the hole. This process is much faster than any heating. It selects out a small number of resonant electrons from the trapped electron distribution to form compounds of at least two electrons resulting in a cold electron compound component. In the long term, when collisionless heating sets in (due to phase mixing in the hole-trapped Buneman wave spectrum), the compounds should also participate in the heating becoming destroyed (due to internal oscillations excited by the higher external temperature) when the compound temperature $T > 2e\Phi$ exceeds the potential of attraction forcing the compound electrons to join back into the trapped population. From this point of view compound formation of electrons in holes will occur preferably in the initial state of the hole before the heating phase sets in. It is thus questionable whether the cold trapped component will survive at all. On the other hand, attractive potentials could as well be generated at later times if only wave modes remain trapped and survive after phase mixing. Hence, the case remains unclear.

In all these scenarios the possibly generated compound plasmas turn out to be of low temperature. Classical "pairing" in collisionless plasma is a non-radiative cooling mechanism acting on a small number of resonant plasma electrons being sensitive to the attractive potential.

3.3 Secondary electron-acoustic wave excitation

The first side-effect of cooling is that the plasma after "pairing" consists of a two-temperature electron plasma of constant charge-to-mass ratio and cold particle density less than plasma density. Such a two-electron temperature plasma excites high frequency/high velocity electron-acoustic waves which are radiated away from the coagulation region. In principle the electron-acoustic waves could be observed if excitation is strong enough to overcome the strong damping of the electron-acoustic waves.

3.4 Electron re-magnetisation in Buneman waves

In the case of the Buneman instability, coagulations form from the slower electron component (Fig. 3) with attractive fields being exponentially damped on large scales. Hence attraction will preferentially be relevant inside BGK-mode electron holes affecting the trapped electron component in the cavity/hole which forms when the Buneman mode evolves nonlinearly. The coagulations constitute a low-density electron population of temperature substantially below T_e which remains hole trapped, unable to escape.

It is interesting to speculate on the importance of this kind of Buneman-induced compound formation in reconnection. Guide field simulations and observations strongly suggest that the Buneman mode causes generation of electron holes during reconnection (Drake et al., 2003; Cattell et al., 2005). In the geomagnetic tail reconnection region, electron temperatures are lower than ion temperatures inhibiting ion-acoustic wave excitation. Electrons in this case are nonmagnetic inside the electron diffusion reconnection site (electron exhaust) being accelerated in the cross-tail field. In presence of a guide field this acceleration causes high guide field aligned velocities exceeding the thermal electron speed, a situation favouring the excitation of Buneman modes and generation of chains of electron holes along the guide field.

Production of a surviving cold dilute compound-electron plasma in the Buneman excited electron holes in the ion-diffusion region and near the reconnection site implies re-magnetisation of the hole-trapped nonmagnetic electrons until their gyroradius r_e^{com} drops below the inertial scale of the plasma. This is easily confirmed by forming the ratio of the compound gyroradius to the bulk electron inertial scale $\lambda_e = c/\omega_e$. Accounting for the constancy of the compound charge-to-mass ratio, this ratio can be written as

$$\frac{r_e^{\mathrm{com}}}{\lambda_e} \approx \left(\beta_e \frac{T_{\mathrm{com}}}{T_e}\right)^{\frac{1}{2}}, \tag{52}$$

where $\beta_e = 2\mu_0 N T_e / B^2$ is the bulk plasma electron-β. Since T_{com}/T_e is substantially less than one, the compound electrons regain magnetisation in the reconnection electron exhaust where the bulk electrons remain to be nonmagnetic, an effect which necessarily affects the evolution of reconnection in several ways. One effect is that magnetised electrons transport magnetic flux into the bulk-electron diffusion region thereby enhancing reconnection. Their stronger magnetisation also modifies reconnection. Moreover, electron holes forming chains along the guide field naturally contribute to amplification and deformation of the guide field on the spatial scale of the holes, a process which self-consistently generates localised non-zero B_z components in the current sheet centre. However, because of the expected very low number of compounds formed, the effect will be rather small if not completely negligible.

In summary, though attractive potentials will certainly arise in various wave particle interactions in plasma, the number of electrons which may under favourable circumstances coagulate and cool down to low temperatures will in all cases be very small and therefore ineffective for plasma processes. Unfortunately, attraction though a natural process does not provide any natural mechanism of large macro-particle number generation. It would be interesting to investigate whether particle "bunching" in low-frequency electromagnetic waves (whistlers, kinetic Alfvén waves, etc.) might be another option of imposing a common dynamic behaviour on large numbers of electrons to perform correlated dynamics and appear as macro-particles. Observation of very dilute cold electrons in the presence of high levels of plasma wave activity would, however, indicate ongoing attraction and coagulation.

Appendix A: No macro-quantum condensation effects

Here we demonstrate that the classical condensate will not undergo any quantum condensation (i.e. it will not become a Bose–Einstein macro-condensate) inside a caviton. The lowest reachable energy level is at the bottom of the caviton. This can be taken as zero-energy level for the composed electrons. Furthermore, composition temperatures are low, the order of a fraction of an eV in the classical condensate. Assuming that the composed electrons obey a Bose distribution in the caviton potential ϕ we thus write for their density

$$\frac{\mathrm{d}n_{\mathrm{com}}}{\mathrm{d}\epsilon} = \frac{1}{4\pi^2}\left(\frac{4m_{\mathrm{e}}}{\hbar^2}\right)^{\frac{3}{2}} \frac{\epsilon^{\frac{1}{2}}}{\mathrm{e}^{\beta(\epsilon-2e\phi-\mu)}-1}. \tag{A1}$$

The chemical potential $\mu \lesssim 0$ is compensated by the caviton potential at inverse temperature β_0 which is calculated from the total density of the trapped particles. The upper limit of the integral can be assumed at infinity. Hence (Fetter and Walecka, 1971)

$$T_0 \equiv \beta_0^{-1} \approx 1.6\frac{\hbar^2}{m_{\mathrm{e}}} n_{\mathrm{com}}^{\frac{2}{3}} \sim 10^{-19} n_{\mathrm{com}}^{\frac{2}{3}}\,\mathrm{eV}. \tag{A2}$$

Since any densities are very low in space plasmas this limit temperature on any Bose–Einstein condensation is practically zero in comparison to the estimated lowest compound temperatures $T_{\mathrm{com}} \sim 0.1\,\mathrm{eV}$. This precludes that in ordinary space plasmas any composed electrons produced would form macro-quantum Bose–Einstein condensates, indeed an intuitive reasoning. Under the extreme conditions in the plasma of neutron star crusts with their high nuclear densities such condensates could possibly occur if Buneman modes would evolve along the neutron star magnetic field.

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